NUMERICAL SIMULATION OF MULTI-DROPLET EVAPORATIVE COOLING

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ABSTRACT

A theoretical study is carried out to predict the thermal behavior of a solid surface subjected to multi-droplet evaporative cooling. A single-droplet numerical code, which has been previously presented and validated, is used here to gain insight into the behavior of a surface subjected to dropwise evaporative cooling. On the basis of the single-droplet results, a generalized model is presented and a novel numerical code is formulated, which analyzes the effects of a multi-droplet evaporative transient on a low thermal conductivity solid surface. The main parameters that characterize the evaporative transient behavior are identified. Some numerical results obtained with this new model are presented and discussed.

INTRODUCTION

Dropwise evaporative cooling of a hot surface is a widely used technique in many engineering applications. For instance, it is commonly found in metallurgical, nuclear and electronic industries. Recently, it has also been used in the space station thermal control systems for equipment cooling.

The single-droplet cooling of hot surfaces has been studied extensively by many researchers. Numerous experimental works provide the basic understanding of the droplet cooling phenomena. Great emphasis has been given to the case of high solid surface temperature where nucleate and film boiling are the dominant heat transfer modes. Makino and Michiyoshi [1][2][3] provide the full range of the boiling curve for the evaporation of a water droplet on a heated surface. The heat transfer rate in the nucleate, transition and film boiling regions is presented as a function of the solid surface superheat. Detailed observations of evaporation time versus initial solid surface temperature, time averaged heat flux and transient temperature distribution are also provided by these authors. In the film boiling region, Seki [4] describes the solid-liquid thermal interaction and measures the rapid change of surface temperature during the formation of the vapor layer below the liquid drop. Contributions concerning the effect of the droplet impact momentum are provided in the experimental studies conducted by Pedersen [5]. These experiments show that the approaching velocity is the dominant parameter in the droplet heat transfer during the film boiling process, while the surface temperature has little effect on it.

For the case of low surface temperatures, experimental data have been collected by diMarzo and Evans [6][7] with photographic inspection of evaporating droplets, and by

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Klassen *et al.* [8] with infrared thermographic techniques. In these works, the influence of the solid thermal properties is emphasized, and predictions are made for materials spanning over two orders of magnitude in terms of thermal conductivity, i.e. from Macor (k = 1.29 W/m°C) to aluminum (k = 180 W/m°C).

In the more general field of spray cooling, early works by Toda [9] and Bonacina *et al.* [10] give a first insight into the phenomena associated with a complex multi-droplet scenario. Their research provides measurements of the heat transfer in mist cooling for low values of the solid surface superheat. Bonacina *et al.* also developed a mathematical model to correlate the experimental data. In all these models, it is pointed out that, at low temperatures, nucleate and film boiling can be neglected since heat conduction through the liquid layers is the dominant heat transfer mechanism.

Rizza [11] provides a numerical investigation for spray evaporation on a semi-infinite solid. A two dimensional transient conduction equation is solved for the solid alone under the assumption of constant surface temperature as a boundary condition. The results from a single-droplet model are used to develop the spray evaporation cooling model. This analysis shows that, in spray cooling, the fraction of the heat transfer surface area on the hot solid covered by the droplets is relatively small.

Among the latest contributions in the field of numerical simulation of dropwise evaporation, a simple model for single-droplet evaporation has been introduced by diMarzo and Evans [7]. It describes the evaporation at the liquid-vapor interface under complete suppression of nucleate boiling and calculates the solid surface temperature distribution with satisfactory accuracy, as confirmed by the comparison between numerical predictions and experimental data (diMarzo and Evans [7], diMarzo *et al.* [12], Tartarini *et al.* [13]). The validated code based on this model is used in the present work to obtain the single-droplet results which lead to the formulation of a comprehensive multi-droplet model for evaporative spray cooling.

MODEL FORMULATION

In order to evaluate the thermal transient due to dropwise evaporation on a hot solid surface, the transient heat conduction equation has to be solved for the liquid and the solid simultaneously and the rate of change of the droplet volume has to be calculated step by step during the entire process. A numerical procedure based on the Boundary Element Method (BEM) for the solid domain and on the Control Volume Method (CVM) for the liquid domain is used to formulate a single-droplet code whose characteristics and validation are extensively described by Tartarini *et al.* [13].

The single-droplet model provides the evaporation time and the solid surface temperature distribution for a droplet evaporating on non-porous materials with thermal conductivity ranging over more than two orders of magnitude. Experiments were conducted on Macor (non-porous, glass-like material, k = 1.29 W/m°C) and aluminum (k = 180 W/m°C), and the results were compared with the numerical predictions. The excellent agreement between experimental data and numerical simulation allows one to use the single-droplet code as an effective basis to build up the more general and complex multi-droplet code.

The transient conduction equation for the liquid and the solid is:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \qquad (1)$$

In analogy with the single-droplet model, the multi-droplet initial conditions can be expressed as:

$$T = T_0 - \frac{q_0}{k_s} z \qquad at \quad t = 0 \tag{2}$$

In a multi-droplet situation, however, the value of the initial surface temperature is a function of the deposition time and location for each droplet. This condition can be written as:

$$T = T_{i_{so}} \qquad i\text{-}th \ \ droplet; \qquad t = t_{i0}; \qquad i = 1,...,N \tag{3}$$

where N is the total number of droplets deposited on the solid surface. This implies that the initial surface temperature T_{is0} must be considered uniform in the small region where the droplet is deposited. Such an assumption is reasonable if both the droplet size and the surface temperature gradient in the deposition region are small. The surface temperature profiles obtained from the single-droplet model show that large temperature gradients on the solid surface occur only near the droplet edge.

For each droplet, let T_{is0} be the uniform initial surface temperature and U_i the transformed temperature, which is defined as:

$$U_i = T_i - T_{i_{so}} + \frac{q_0}{k_s} z \tag{4}$$

From the single-droplet model, one can write the governing equation and the associated boundary and initial conditions for the i-th droplet as follows:

$$\frac{\partial U_i}{\partial t} = \alpha \nabla^2 U_i \qquad\qquad U_i = 0 \quad at \quad t = t_{io}$$

$$\frac{\partial U_i}{\partial r} = 0 \quad at \quad r \to \infty \qquad \frac{\partial U_i}{\partial z} = 0 \quad at \quad z \to \infty$$

$$k_s \left(\frac{\partial U_i}{\partial z} \right)_s = k_l \left(\frac{\partial U_i}{\partial z} \right)_l \qquad at \quad z = 0, \ r \le R$$

$$k_s \frac{\partial U_i}{\partial z} = h U_i \qquad at \quad z = 0, \ r > R$$

(5,6,7,8,9,10)

The formulation of the multi-droplet model is based on the partition of the droplet cooling domain into a near-field and a far-field. In such a way, the near-field temperature distribution can be obtained from the single-droplet calculations (which are compiled in a data base), while the far-field temperature distribution is obtained from a closed form solution. The solid surface temperature is then calculated by superimposing the values from the data base or the closed form solution.

In the near-field the single-droplet results are directly applied. A typical temperature distribution from the droplet center to the far-field at different times during the evaporative transient is shown in Fig. 1. When the distance from the droplet is large enough, the cooling effect propagation time becomes greater than the droplet

evaporation time. On Macor, at a distance r equal to ten times the droplet radius (i.e. $r \approx 1$ cm) this propagation time can be estimated as:

$$t \sim \frac{r^2}{\alpha_s} \sim \frac{10^{-4}}{10^{-7}} = 1000\ s \tag{11}$$

while the evaporation times for the ranges of values mentioned above span between 10 and 100 seconds.

Therefore, in the far-field (for distances greater than 10 droplet radii) the surface temperature distribution can be calculated by a closed form solution (by Carslaw and Jaeger [14]), which approximates the droplet transient behavior as an instantaneous point sink. That is:

$$\Delta T = T - T_{s0} = \frac{Q}{8(\pi \alpha_s t)^{3/2}} e^{-\frac{r^2}{4\alpha_s t}} \tag{12}$$

where the "strength" of the instantaneous point sink Q is:

$$Q = -2 V_0 \frac{\rho_l}{\rho_s} \frac{\Lambda}{c_{p_s}} \tag{13}$$

This instantaneous point sink is generated at time t, while for the evaporation of a single droplet the heat removal is spread over the evaporation time τ. By calculating the spatial averaged heat flux and its mean value over the evaporation time (see Fig. 2), one can find that the instantaneous point sink can be set at a time t equal to 60 percent of the evaporation time τ for Macor. This corresponds to a delay of $0.6\ \tau$ in the deposition time. Therefore, for each droplet, the expression of the temperature difference in the far-field points is given by:

$$\Delta T = T - T_{s0} = -\frac{V_0 \frac{\rho_l}{\rho_s} \frac{\Lambda}{c_{p_s}}}{4[\pi \alpha_s (t - 0.6\tau)]^{\frac{3}{2}}} e^{-\frac{r^2}{4\alpha_s (t - 0.6\tau)}} \tag{14}$$

Figure 3 shows the difference between the temperature calculated by the single-droplet model and the closed form solution at different times as a function of the radial distance for Macor. The error is within an acceptable range for $r/R \geq 10$. Figure 4 shows the same temperature difference calculated at $r/R = 10$ as a function of time for three different cases of initial surface temperature. One can see that the difference is always bound within less than ± 0.1°C.

Due to the linearity of the conduction equation, the principle of superposition can be invoked in the multi-droplet model, and one can write:

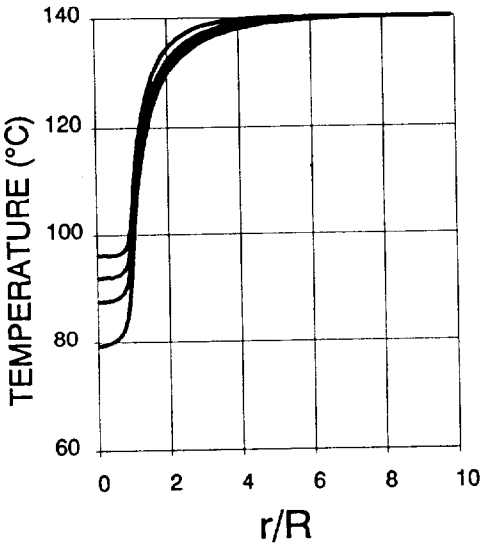

FIGURE 1 - Single-droplet model: Typical solid surface temperature profiles during the evaporative transient (T_{s0} = 140 °C; t/τ = 0.3, 0.5, 0.7 and 0.9 top-to-bottom; V_0 = 10 μl on Macor)

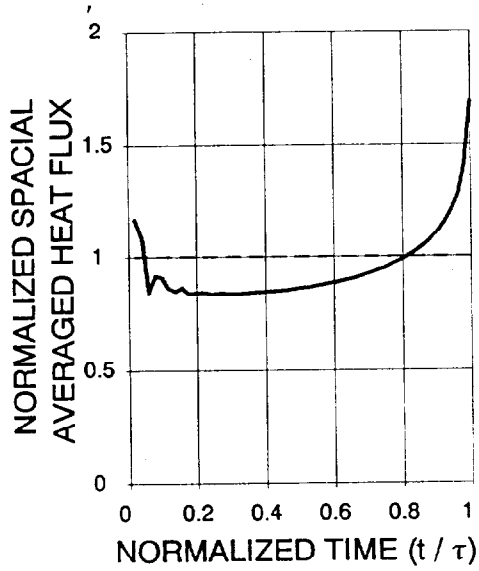

FIGURE 2 - Single-droplet model: Normalized heat flux at the solid-liquid interface versus normalized time for a 10 μl drop evaporating on Macor at T_{s0} = 140 °C.

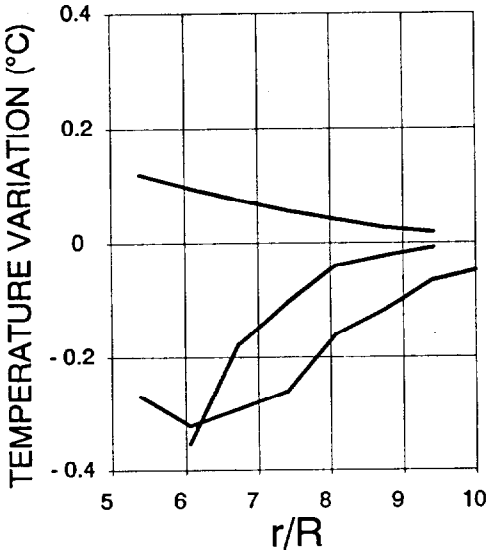

FIGURE 3 - Spatial behavior of the temperature difference between single-droplet model calculations and closed form solution.

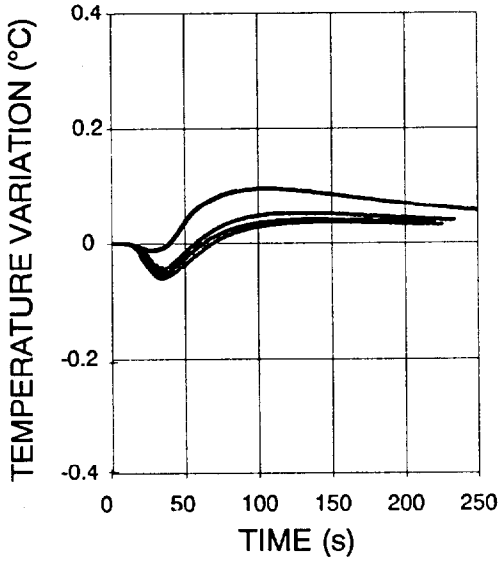

FIGURE 4 - Temporal behavior of the temperature difference between single-droplet model calculations and closed form solution.

$$U = \sum_{i=1}^{N} U_i \tag{15}$$

The transformed temperature U, as defined in Eq.(15), is the solution of the multi-droplet cooling problem. Thus the temperature T can be expressed as:

$$T = T_{s0} + \sum_{i=1}^{N} \left(T_i - T_{i_{s0}} \right) \tag{16}$$

The droplet distribution pattern on the solid surface is determined by a simple numerical program. It simulates the action of a dispenser that releases droplets of fixed volume with assigned frequency at random locations. A typical droplet pattern is shown in Fig. 5. Given this distribution pattern, all cooling effects from each droplet are superimposed. The summation provides a detailed temperature distribution as a function of time in a multi-droplet evaporative cooling transients.

The most useful parameter to describe the temperature distribution in multi-droplet cooling is a spatial average temperature of the calculation domain which is defined as follows:

$$T_{ave} = \frac{\iint T\,dx\,dy}{\iint dx\,dy} = \frac{\sum_{i=1}^{N_{pi}} \sum_{j=1}^{N_{pj}} T_{ij}\,\Delta x_i \Delta y_j}{\sum_{i=1}^{N_{pi}} \sum_{j=1}^{N_{pj}} \Delta x_i \Delta y_j} \tag{17}$$

Further simplification can be made if the size of every mesh is the same. Then the spatial average temperature can be interpreted as the summation of the temperatures of all the specified points on the surface divided by the total number of the grid points N_p:

$$T_{ave} = \frac{\sum_{k=1}^{N_p} T_k}{N_p} \tag{18}$$

The controlling parameters for the multi-droplet cooling include the initial solid surface temperature, the properties of the solid and the mass flux of the liquid spray. Even if only Macor has been used during this study, any other material can be considered by inserting the appropriate data base. The only limit to the initial surface temperature is set by the operating condition of full suppression of nucleate boiling (for Macor the maximum temperature that can be reached before the onset of nucleate boiling is about 160°C). The liquid mass flux is defined as:

$$\dot{m} = \frac{\rho f V_0}{A} \tag{19}$$

and, given a fixed droplet volume and the liquid density, it is a function of the droplet deposition frequency f and of the spray impingement area A.

RESULTS AND DISCUSSION

The main objective of a spray cooling analysis is to evaluate the temperature distribution on the solid surface as a function of time during the multi-droplet evaporative process. As previously described, the temperature at any point of the solid surface can be obtained by superimposing the results of many single-droplet calculations. Given an initial solid surface temperature and a pre-determined droplet distribution pattern, the temperature distribution at any selected time can be predicted by the code.

Figures 6 to 8 show some three-dimensional temperature plots for a portion of the solid surface at different times. These figures pertain to the transient with an initial solid surface temperature of 160°C and a liquid mass flux of $3x10^{-4}$ kg/m^{2}s. At the beginning of the transient, the region influenced by one droplet is easily identifiable. The temperature variation is very sharp in the area where the droplet is evaporating. When more droplets are deposited on the solid surface, the temperature at any point is the result of the cooling effect caused by all the droplets and the single droplet contribution becomes less evident.

Figures 9 and 10 show the average solid surface temperature versus time in two typical cases analyzed by the multi-droplet model. The fluctuations of the average temperature are related with the random distribution pattern of the droplets on the solid surface. These plots show a rapid decrease in the initial phase of the process. Note that the evaporation time on Macor given the initial conditions of the reported cases ranges from 25 to 100 seconds. During this time interval, the solid surface is constantly cooled down by the evaporating droplets, and the average temperature steadily decreases. When a droplet is completely evaporated, the corresponding surface points show a temperature recovery. The initial volume of the droplets used here is constant ($V_0=5\mu l$).

CONCLUSIONS

A dropwise evaporation model is presented, which calculates the transient temperature distribution on a hot solid surface subjected to the cooling effect of an evaporating water spray. The numerical simulation can be applied to a broad range of solid non-porous materials. A single-droplet model, provides accurate predictions of the droplet evaporation time, of the temperature distribution and of the heat fluxes inside the droplet and on the solid surface. These results are used as a basis for the formulation of the multi-droplet numerical code.

By applying the linear characteristic of the conduction equation, the principle of superposition is used in the multi-droplet model. The combination of the thermal effects due to a pre-determined droplet pattern generates surface temperature distributions as a function of time.

A spatial average temperature is used to describe the cooling mechanism on the solid surface. The spray mass flow rate and the initial solid temperature are identified as the governing parameters in the multi-droplet evaporation process.

Further work to evaluate the influence of different surface materials on the evaporative spray cooling is in progress.

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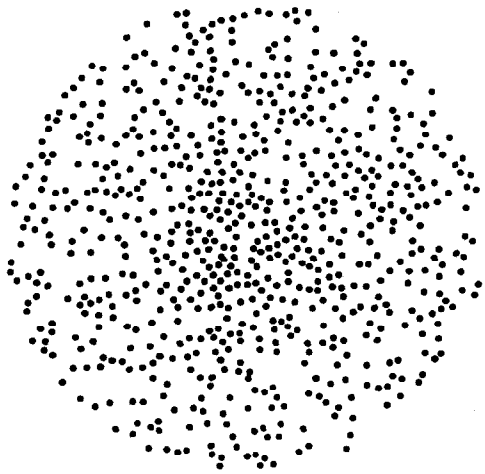

IMPACTED AREA RADIUS = 10 cm

FIGURE 5 - Multi-droplet model: Typical droplet distribution on the solid surface as simulated by the code.

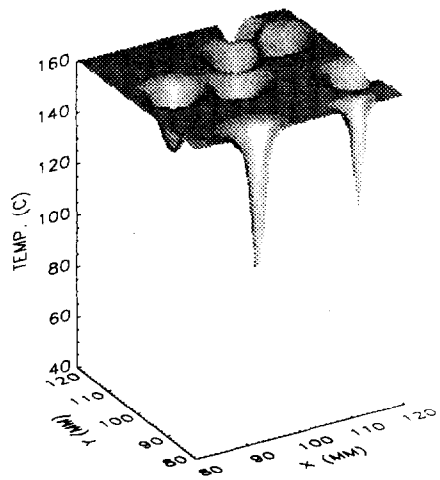

FIGURE 6 - Multi-droplet model: Temperature distribution on the solid surface after 5 seconds (V_0 = 5 μl; T_{s0} = 160 °C; $\dot{m}$ = 0.0016 kg/m^{2}s)

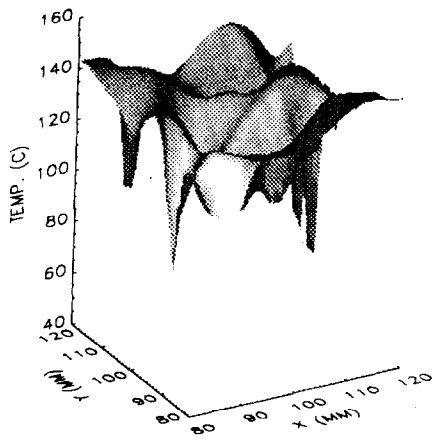

FIGURE 7 - Multi-droplet model: Temperature distribution on the solid surface after 60 seconds (V_0 = 5 μl; T_{s0} = 160 °C; $\dot{m}$ = 0.0016 kg/m^{2}s)

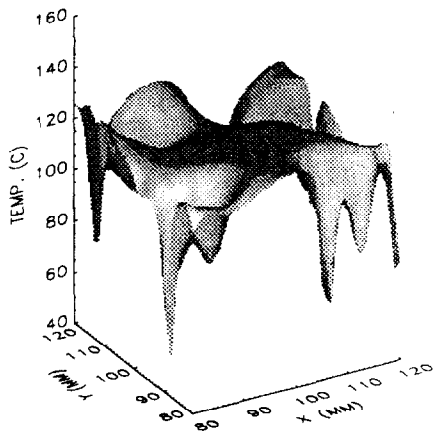

FIGURE 8 - Multi-droplet model: Temperature distribution on the solid surface after 200 seconds (V_0 = 5 μl; T_{s0} = 160 °C; $\dot{m}$ = 0.0016 kg/m^{2}s)

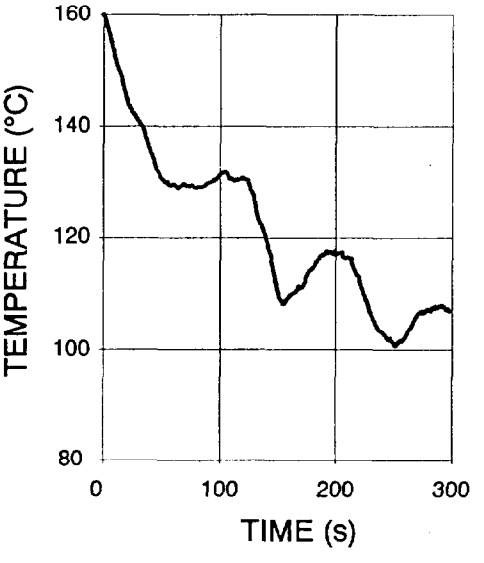

FIGURE 9 - Multi-droplet model: Average solid temperature distribution versus time (V_0 = 5 μl; T_{s0} = 160 °C; $\dot{m}$ = 0.0016 kg/m^{2}s)

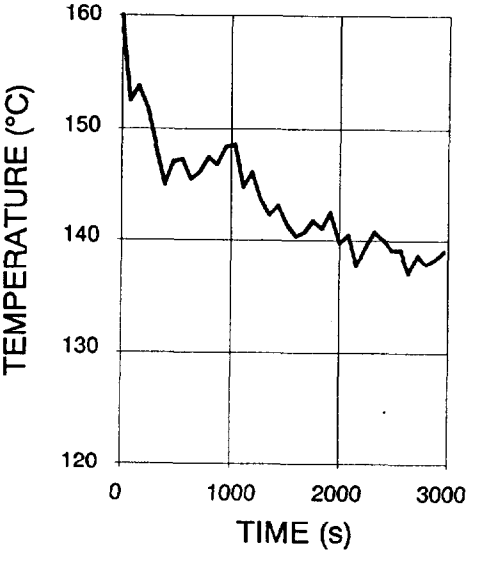

FIGURE 10 - Multi-droplet model: Average solid temperature distribution versus time (V_0 = 5 μl; T_{s0} = 160 °C; $\dot{m}$ = 0.0008 kg/m^{2}s)

NOMENCLATURE

A	spray impingement area
c_p	specific heat
d	droplet distribution density
f	droplet deposition frequency
h	overall heat transfer coefficient
k	thermal conductivity
m	mass flux
N	total number of droplets
N_p	number of grid points
Q	strength of the instantaneous point sink
q	axial heat flux
$\bar{q}$	spatial averaged heat flux
R	radius of the wetted area
r	radial coordinate
T	temperature
t	time
U	transformed temperature
V	droplet volume
x,y	cartesian coordinates
z	axial coordinate

α	thermal diffusivity
Λ	latent heat of vaporization
ρ	density
τ	evaporation time

subscripts

ave	average
l	liquid
s	solid
0	initial

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